CHAPTER 19
NUMERICAL TRIGONOMETRY

The word "trigonometry" means "measurement by triangles." As it is presented in many textbooks, trigonometry includes topics other than triangles and measurement. However, this chapter is intended only as an introduction to the numerical aspects of trigonometry as they relate to measurement of lengths and angles.

SPECIAL PROPERTIES OF RIGHT TRIANGLES

A RIGHT TRIANGLE has been defined as any triangle containing a right angle. The side opposite the right angle in a right triangle is a HYPOTENUSE. (See fig. 19-1.) In figure 19-1, side AC is the hypotenuse.

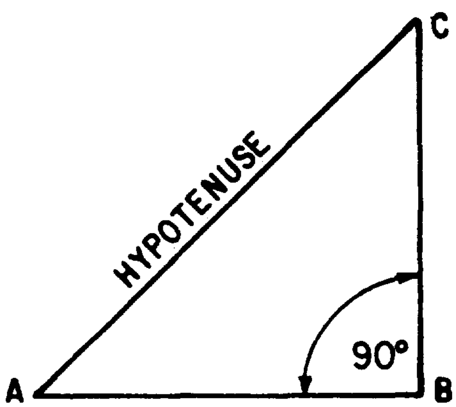

Figure 19-1.—A right triangle.

An important property of all right triangles, which relates the lengths of the three sides, was discovered by the Greek philosopher Pythagoras.

PYTHAGOREAN THEOREM

The rule of Pythagoras, or PYTHAGOREAN THEOREM, states that the square of the length of the hypotenuse (in any right triangle) is equal to the sum of the squares of the lengths of the other two sides. For example, if the sides are labeled as in figure 19-2 (A), the Pythagorean Theorem is stated in symbols as follows:

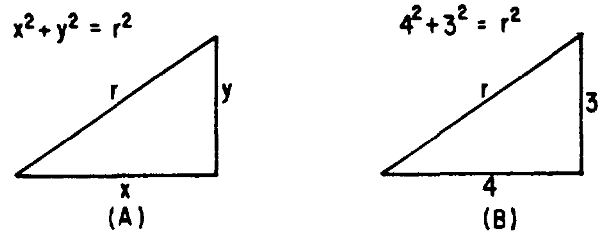

Figure 19-2.—The Pythagorean Theorem. (A) General triangle; (B) triangle with sides of specific lengths.

$$x^2 + y^2 = r^2$$

An example of the use of the Pythagorean Theorem in a problem follows:

EXAMPLE: Find the length of the hypotenuse in the triangle shown in figure 19-2 (B).

SOLUTION:
$$r^2 = 3^2 + 4^2$$
$$r = \sqrt{9 + 16}$$
$$= \sqrt{25} = 5$$

EXAMPLE: An observer on a ship at point A, figure 19-3, knows that his distance from point C is 1,200 yards and that the length of BC is 1,300 yards. He measures angle A and finds that it is 90°. Calculate the distance from A to B.

SOLUTION: By the rule of Pythagoras,
$$(BC)^2 = (AB)^2 + (AC)^2$$
$$(1{,}300)^2 = (AB)^2 + (1{,}200)^2$$
$$(1{,}300)^2 - (1{,}200)^2 = (AB)^2$$
$$(13 \times 10^2)^2 - (12 \times 10^2)^2 = (AB)^2$$
$$(169 \times 10^4) - (144 \times 10^4) = (AB)^2$$
$$25 \times 10^4 = (AB)^2$$
$$500 \text{ yd} = AB$$

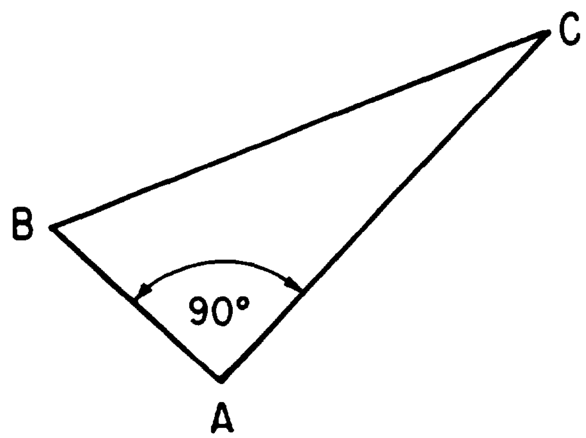

Figure 19-3.—Using the Pythagorean Theorem.

SIMILAR RIGHT TRIANGLES

Two right triangles are SIMILAR if one of the acute angles of the first is equal to one of the acute angles of the second. This conclusion is supported by the following reasons:

1. The right angle in the first triangle is equal to the right angle in the second, since all right angles are equal.
2. The sum of the angles of any triangle is 180°. Therefore, the sum of the two acute angles in a right triangle is 90°.
3. Let the equal acute angles in the two triangles be represented by A and A' respectively. (See fig. 19-4.) Then the other acute angles, B and B', are as follows:

$$B = 90° - A$$
$$B' = 90° - A'$$

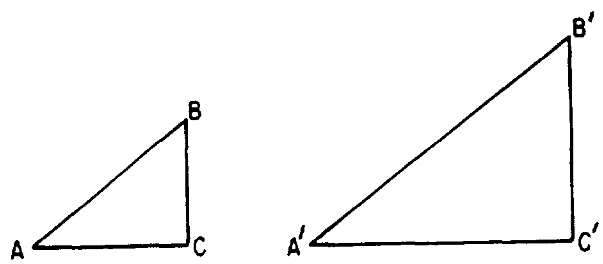

Figure 19-4.—Similar right triangles.

4. Since angles A and A' are equal, angles B and B' are also equal.
5. We conclude that two right triangles with one acute angle of the first equal to one acute angle of the second have all of their corresponding angles equal. Thus the two triangles are similar.

Practical situations frequently occur in which similar right triangles are used to solve problems. For example, the height of a tree can be determined by comparing the length of its shadow with that of a nearby flagpole, as shown in figure 19-5.

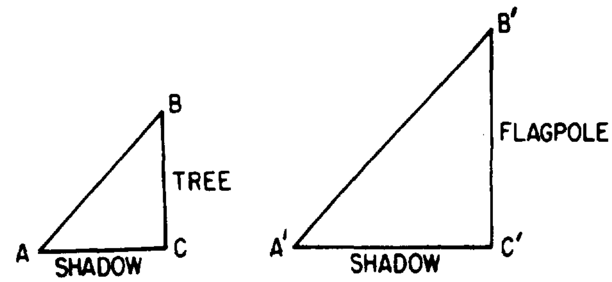

Figure 19-5.—Calculation of height by comparison of shadows.

Assume that the rays of the sun are parallel and that the tree and flagpole both form 90° angles with the ground. Then triangles ABC and A'B'C' are right triangles and angle B is equal to angle B'. Therefore, the triangles are similar and their corresponding sides are proportional, with the following result:

$$\frac{BC}{AC} = \frac{B'C'}{A'C'}$$

$$BC = \frac{(AC) \times (B'C')}{A'C'}$$

Suppose that the flagpole is known to be 30 feet high, the shadow of the tree is 12 feet long, and the shadow of the flagpole is 24 feet long. Then

$$BC = \frac{12 \times 30}{24} = 15 \text{ feet}$$

Practice problems.

1. A mast at the top of a building casts a shadow whose tip is 48 feet from the base of the building. If the building is 12 feet high and its shadow is 32 feet long, what is the length of the mast? (NOTE: If the length of the mast is x, then the height of the mast above the ground is x + 12.)

2. Figure 19-6 represents an L-shaped building with dimensions as shown. On the line of sight from A to D, a stake is driven at C, a point 8 feet from the building and 10 feet from A. If ABC is a right angle, find the length of AB and the length of AD. Notice that AE is 18 feet and ED is 24 feet.

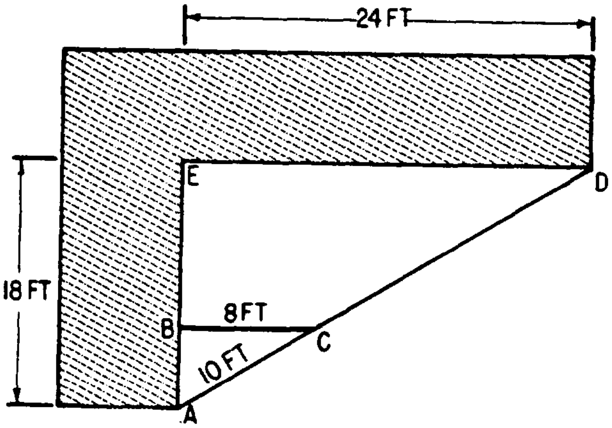

Figure 19-6.—Using similar triangles.

Answers:

1. 6 feet

2. AB = 6 feet
   AD = 30 feet

TRIGONOMETRIC RATIOS

The relationships between the angles and the sides of a right triangle are expressed in terms of TRIGONOMETRIC RATIOS. For example, in figure 19-7, the sides of the triangle are named in accordance with their relationship to angle θ. In trigonometry, angles are usually named by means of Greek letters. The Greek name of the symbol θ is theta.

The six trigonometric ratios for the angle θ are listed in table 19-1.

The ratios are defined as follows:

1. $\sin\theta = \dfrac{\text{side opposite }\theta}{\text{hypotenuse}} = \dfrac{y}{r}$

2. $\cos\theta = \dfrac{\text{side adjacent to }\theta}{\text{hypotenuse}} = \dfrac{x}{r}$

3. $\tan\theta = \dfrac{\text{side opposite }\theta}{\text{side adjacent to }\theta} = \dfrac{y}{x}$

4. $\cot\theta = \dfrac{\text{side adjacent to }\theta}{\text{side opposite }\theta} = \dfrac{x}{y}$

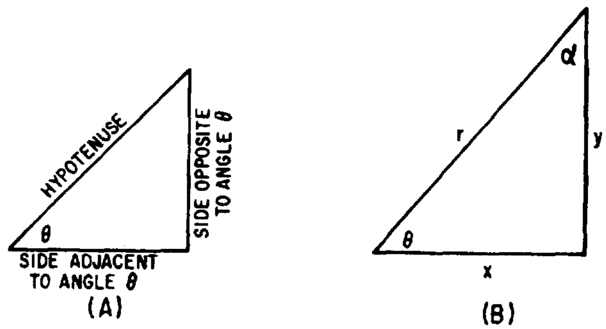

Figure 19-7.—Relationship of sides and angles in a right triangle. (A) Names of the sides; (B) symbols used to designate the sides.

Table 19-1.—Trigonometric ratios.

Name of ratio	Abbreviation
sine of θ	$\sin\theta$
cosine of θ	$\cos\theta$
tangent of θ	$\tan\theta$
cotangent of θ	$\cot\theta$
secant of θ	$\sec\theta$
cosecant of θ	$\csc\theta$

5. $\sec\theta = \dfrac{\text{hypotenuse}}{\text{side adjacent to }\theta} = \dfrac{r}{x}$

6. $\csc\theta = \dfrac{\text{hypotenuse}}{\text{side opposite to }\theta} = \dfrac{r}{y}$

The other acute angle in figure 19-7 (B) is labeled α (Greek alpha). The side opposite α is x and the side adjacent to α is y. Therefore the six ratios for α are as follows:

1. $\sin\alpha = \dfrac{x}{r}$
2. $\cos\alpha = \dfrac{y}{r}$
3. $\tan\alpha = \dfrac{x}{y}$
4. $\cot\alpha = \dfrac{y}{x}$
5. $\sec\alpha = \dfrac{r}{y}$
6. $\csc\alpha = \dfrac{r}{x}$

Suppose that the sides of triangle (B) in figure 19-7 are as follows: $x = 3$, $y = 4$, $r = 5$. Then each of the ratios for angles θ and α may be expressed as a common fraction or as a decimal. For example,

$$\sin \theta = \frac{4}{5} = 0.800$$

$$\sin \alpha = \frac{3}{5} = 0.600$$

Decimal values have been computed for ratios of angles between 0° and 90°, and values for angles above 90° can be expressed in terms of these same values by means of conversion formulas. Appendix II of this training course gives the sine, cosine, and tangent of angles from 0° to 90°. The secant, cosecant, and cotangent are calculated, when needed, by using their relationships to the three principal ratios. These relationships are as follows:

$$\text{secant } \theta = \frac{1}{\text{cosine } \theta}$$

$$\text{cosecant } \theta = \frac{1}{\text{sine } \theta}$$

$$\text{cotangent } \theta = \frac{1}{\text{tangent } \theta}$$

TABLES

Tables of decimal values for the trigonometric ratios may be constructed in a variety of ways. Some give the angles in degrees, minutes, and seconds; others in degrees and tenths of a degree. The latter method is more compact and is the method used for appendix II. The "headings" at the bottom of each page in appendix II provide a convenient reference showing the minute equivalents for the decimal fractions of a degree. For example, 12' (12 minutes) is the equivalent of 0.2°.

Finding the Function Value

The trigonometric ratios are sometimes called FUNCTIONS, because the value of the ratio depends upon (is a function of) the angle size. Finding the function value in appendix II is easily accomplished. For example, the sine 35° is found by looking in the "sin" row opposite the large number 35, which is located in the extreme left-hand column.

Since our angle in this example is exactly 35°, we look for the decimal value of the sine in the column with the 0.0° heading. This column contains decimal values for functions of the angle plus 0.0°; in our example, 35° plus 0.0°, or simply 35.0°. Thus we find that the sine of 35.0° is 0.5736. By the same reasoning, the sine of 42.7° is 0.6782, and the tangent of 32.3° is 0.6322.

A typical problem in trigonometry is to find the value of an unknown side in a right triangle when only one side and one acute angle are known. EXAMPLE: In triangle ABC (fig. 19-8), find the length of AC if AB is 13 units long and angle CAB is 34.7°.

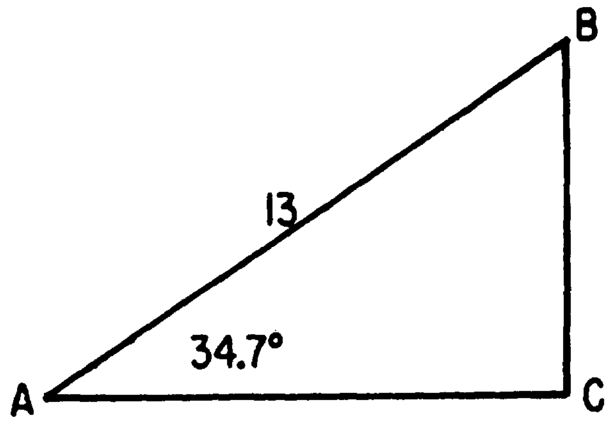

Figure 19-8.—Using the trigonometric ratios to evaluate the sides.

SOLUTION:

$$\frac{AC}{13} = \cos 34.7°$$

$$AC = 13 \cos 34.7°$$

$$= 13 \times 0.8221$$

$$= 10.69 \text{ (approx.)}$$

The angles of a triangle are frequently stated in degrees and minutes, rather than degrees and tenths. For example, in the foregoing problem, the angle might have been stated as 34°42'. When the stated number of minutes is an exact multiple of 6 minutes, the minute entries at the bottom of each page in appendix II may be used.

Finding the Angle

Problems are frequently encountered in which two sides are known, in a right triangle, but neither of the acute angles is known. For example, by applying the Pythagorean Theorem we can verify that the triangle in figure 19-9 is

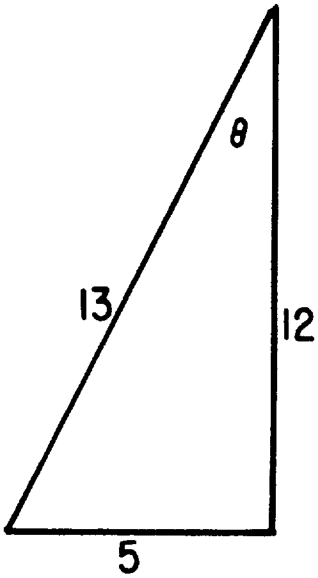

Figure 19-9.—Using trigonometric ratios to evaluate angles.

a right triangle. The only information given, concerning angle θ, is the ratio of sides in the triangle. The size of θ is calculated as follows:

$$\tan \theta = \frac{5}{12} = 0.4167$$

$$\theta = \text{the angle whose tangent is } 0.4167$$

Assuming that the sides and angles in figure 19-9 are in approximately the correct proportions, we estimate that angle θ is about 20°. The table entries for the tangent in the vicinity of 20° are slightly too small, since we need a number near 0.4167. However, the tangent of 22° 36' is 0.4163 and the tangent of 22°42' is 0.4183. Therefore, θ is between 22° 36' and 22°42'.

Interpolation

It is frequently necessary to estimate the value of an angle to a closer approximation than is available in the table. This is equivalent to estimating between table entries, and the process is called INTERPOLATION. For example, in the foregoing problem it was determined that the angle value was between 22°36' and 22°42'. The following paragraphs describe the procedure for interpolating to find a closer approximation to the value of the angle.

The following arrangement of numbers is recommended for interpolation:

$$\begin{array}{ccc}
 & \text{ANGLE} & \text{TANGENT} \\
6' \left\{ \begin{array}{c} 22°36' \\ \theta \\ \\ 22°42' \end{array} \right. & & \left. \begin{array}{c} \left. \begin{array}{c} 0.4163 \\ 0.4167 \end{array} \right\} .0004 \\ \\ 0.4183 \end{array} \right\} .0020
\end{array}$$

The spread between 22°36' and 22°42' is 6', and we use the comparison of the tangent values to determine how much of this 6' spread is included in θ, the angle whose value is sought. Notice that the tangent of θ is different from tan 22°36' by only 0.0004, and the total spread in the tangent values is 0.0020. Therefore, the tangent of θ is $\frac{0.0004}{0.0020}$ of the way between the tangents of the two angles given in the table. This is 1/5 of the total spread, since

$$\frac{0.0004}{0.0020} = \frac{4}{20} = \frac{1}{5}$$

Another way of arriving at this result is to observe that the total spread is 20 ten-thousandths, and that the partial spread corresponding to angle θ is 4 ten-thousandths. Since 4 out of 20 is the same as 1 out of 5, angle θ is 1/5 of the way between 22°36' and 22°42'.

Taking 1/5 of the 6' spread between the angles, we have the following calculation:

$$\frac{1}{5} \times 6' = \frac{1}{5} \times 5'60''$$

$$= 1'12'' \text{ (1 minute and 12 seconds)}$$

The 12" obtained in this calculation causes our answer to appear to have greater accuracy than the tables from which it is derived. This apparent increase in accuracy is a normal result of interpolation. Final answers based on interpolated data should be rounded off to the same degree of accuracy as that of the original data.

The value of 1 minute and 12 seconds found in the foregoing problem is added to 22°36', as follows:

$$\theta = 22°36' + 1'12'' = 22°37'12''$$

Therefore θ is 22°37', approximately.

The foregoing problem could have been solved in terms of tenths and hundredths of a degree, rather than minutes, as follows:

$$
\begin{array}{cccc}
 & \text{ANGLE} & \text{TANGENT} & \\
0.1^\circ \left\{ \begin{array}{c} 22.60^\circ \\ \theta \\ 22.70^\circ \end{array} \right. & & \left. \begin{array}{c} \left. \begin{array}{c} 0.4163 \\ 0.4167 \end{array} \right\} 0.0004 \\ 0.4183 \end{array} \right\} 0.0020 &
\end{array}
$$

In this example, we are concerned with an angular spread of 0.10° and θ is located 1/5 of the way through this spread. Thus we have

$$\theta = 22.60^\circ + \left(\frac{1}{5} \times 0.10^\circ\right)$$

$$\theta = 22.60^\circ + 0.02^\circ$$

$$\theta = 22.62^\circ$$

Interpolation must be approached with common sense, in order to avoid applying corrections in the wrong direction. For example, the cosine of an angle decreases in value as the angle increases from 0° to 90°. If we need the value of the cosine of an angle such as 22°39', the calculation is as follows:

$$
\begin{array}{ccc}
 \text{ANGLE} & & \text{COSINE} \\
6' \left\{ \begin{array}{c} \left. \begin{array}{c} 22^\circ 36' \\ 22^\circ 39' \end{array} \right\} 3' \\ 22^\circ 42' \end{array} \right. & & \left. \begin{array}{c} 0.9232 \\ \\ 0.9225 \end{array} \right\} 0.0007
\end{array}
$$

In this example, it is easy to see that 22°39' is halfway between 22°36' and 22°42'. Therefore the cosine of 22°39' is halfway between the cosine of 22°36' and that of 22°42'. Taking one-half of the spread between these cosines, we then SUBTRACT from 0.9232 to find the cosine of 22°39', as follows:

$$\cos 22^\circ 39' = 0.9232 - \left(\frac{1}{2} \times 0.0007\right)$$

$$= 0.9232 - 0.00035$$

$$= 0.92285$$

$$= 0.9229 \text{ (approximately)}$$

Practice problems:

1. Use the table in appendix II to find the decimal value of each of the following ratios:

a. tan 45°   d. sin 37°14'
b. sin 60°   e. cos 51.5°
c. cos 42°6'   f. tan 13.75°

2. Find the angle which corresponds to each of the following decimal values in appendix II:

a. sin θ = 0.2790   c. tan θ = 0.7604
b. cos θ = 0.9018   d. sin θ = 0.8142

Answers:

1. a. 1   d. 0.6051
   b. 0.8660   e. 0.6225
   c. 0.7420   f. 0.2447

2. a. θ = 16.2°   c. θ = 37°15'
   b. θ = 25°36'   d. θ = 54°30'

RIGHT TRIANGLES WITH SPECIAL ANGLES AND SIDE RATIOS

Three types of right triangles are especially significant because of their frequent occurrence. These are the 30°-60°-90° triangle, the 45°-90° triangle, and the 3-4-5 triangle.

THE 30°-60°-90° TRIANGLE

The 30°-60°-90° triangle is so named because these are the sizes of its three angles. The sides of this triangle are in the ratio of 1 to $\sqrt{3}$ to 2, as shown in figure 19-10.

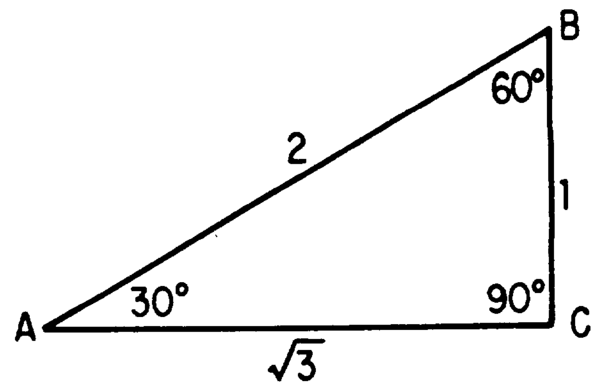

Figure 19-10.—30°-60°-90° triangle.

The sine ratio for the 30° angle in figure 19-10 establishes the proportionate values of the sides. For example, we know that the sine of 30° is 1/2; therefore side AB must be twice as long as BC. If side BC is 1 unit long, then side AB is 2 units long and, by the rule of Pythagoras, AC is found as follows:

$$AC = \sqrt{(AB)^2 - (BC)^2}$$
$$= \sqrt{4 - 1} = \sqrt{3}$$

Regardless of the size of the unit, a 30°-60°-90° triangle has a hypotenuse which is 2 times as long as the shortest side. The shortest side is opposite the 30° angle. The side opposite the 60° angle is $\sqrt{3}$ times as long as the shortest side. For example, suppose that the hypotenuse of a 30°-60°-90° triangle is 30 units long; then the shortest side is 15 units long, and the length of the side opposite the 60° angle is 15 $\sqrt{3}$ units.

Practice problems. Without reference to tables or to the rule of Pythagoras, find the following lengths and angles in figure 19-11:

1. Length of AC.
2. Size of angle A.
3. Size of angle B.
4. Length of RT.
5. Length of RS.
6. Size of angle T.

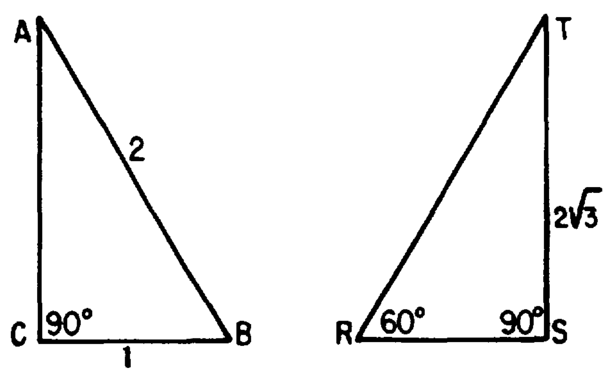

Figure 19-11.—Finding parts of 30°-60°-90° triangles.

Answers:

1. $\sqrt{3}$
2. 30°
3. 60°
4. 4
5. 2
6. 30°

THE 45°-90° TRIANGLE

Figure 19-12 illustrates a triangle in which two angles measure 45° and the third angle measures 90°. Since angles A and B are equal, the sides opposite them are also equal. Therefore, AC equals CB. Suppose that CB is 1 unit long; then AC is also 1 unit long, and the length of AB is calculated as follows:

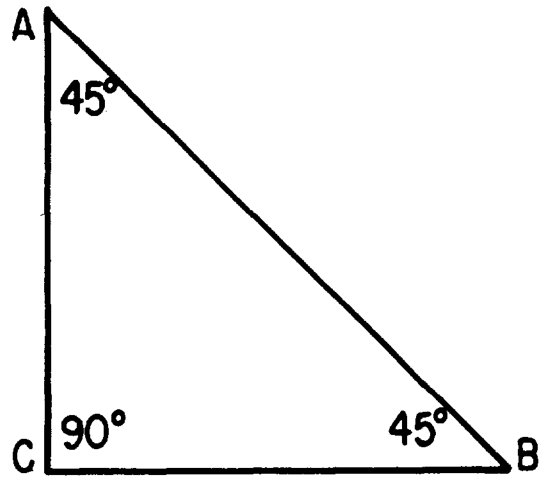

Figure 19-12.—A 45°-90° triangle.

$$(AB)^2 = 1^2 + 1^2 = 2$$
$$AB = \sqrt{2}$$

Regardless of the size of the triangle, if it has two 45° angles and one 90° angle, its sides are in the ratio 1 to 1 to $\sqrt{2}$. For example, if sides AC and CB are 3 units long, AB is 3 $\sqrt{2}$ units long.

Practice problems. Without reference to tables or to the rule of Pythagoras, find the following lengths and angles in figure 19-13:

1. AB
2. BC
3. Angle B

Answers:

1. 2 $\sqrt{2}$
2. 2
3. 45°

THE 3-4-5 TRIANGLE

The triangle shown in figure 19-14 has its sides in the ratio 3 to 4 to 5. Any triangle with its sides in this ratio is a right triangle.

It is a common error to assume that a triangle is a 3-4-5 type because two sides are known to be in the ratio 3 to 4, or perhaps 4 to 5. Figure 19-15 shows two examples of triangles which happen to have two of their sides in the stated ratio, but not the third side. This

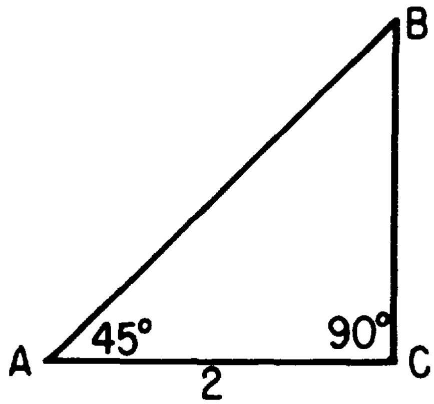

Figure 19-13.—Finding unknown parts in a 45°-90° triangle.

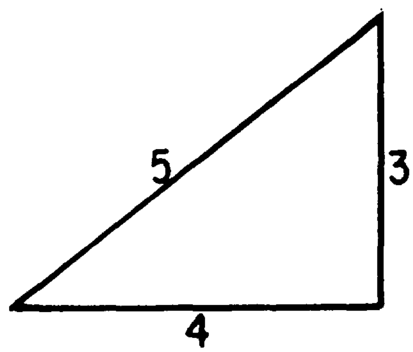

Figure 19-14.—A 3-4-5 triangle.

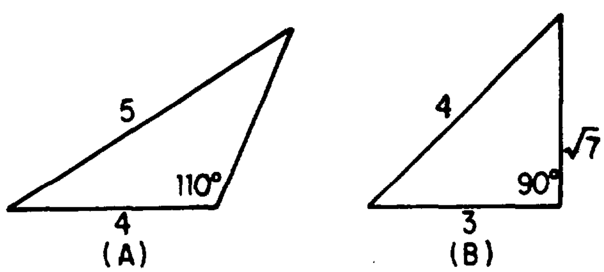

Figure 19-15.—Triangles which may be mistaken for 3-4-5 triangles.

can be because the triangle is not a right triangle, as in figure 19-15 (A). On the other hand, even though the triangle is a right triangle its longest side may be the 4-unit side, in which case the third side cannot be 5 units long. (See fig. 19-15 (B).)

It is interesting to note that the third side in figure 19-15 (B) is $\sqrt{7}$. This is a very unusual coincidence, in which one side of a right triangle is the square root of the sum of the other two sides.

Related to the basic 3-4-5 triangle are all triangles whose sides are in the ratio 3 to 4 to 5 but are longer (proportionately) than these basic lengths. For example, the triangle pictured in figure 19-6 is a 3-4-5 triangle.

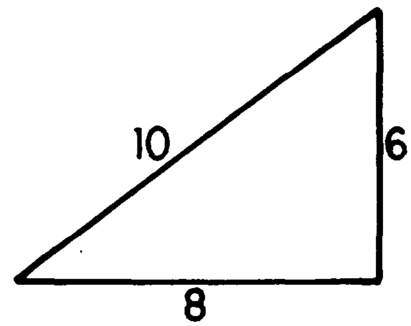

Figure 19-16.—Triangle with sides which are multiples of 3, 4, and 5.

The 3-4-5 triangle is very useful in calculations of distance. If the data can be adapted to fit a 3-4-5 configuration, no tables or calculation of square root (Pythagorean Theorem) are needed.

EXAMPLE: An observer at the top of a 40-foot vertical tower knows that the base of the tower is 30 feet from a target on the ground. How does he calculate his slant range (direct line of sight) from the target?

SOLUTION: Figure 19-17 shows that the desired length, AB, is the hypotenuse of a right triangle whose shorter sides are 30 feet and 40 feet long. Since these sides are in the ratio 3 to 4 and angle C is 90°, the triangle is a 3-4-5 triangle. Therefore, side AB represents the 5-unit side of the triangle. The ratio 30 to 40 to 50 is equivalent to 3-4-5, and thus side AB is 50 units long.

Practice problems. Without reference to tables or to the rule of Pythagoras, solve the following problems:

1. An observer is at the top of a 30-foot vertical tower. Calculate his slant range from a target on the ground which is 40 feet from the base of the tower.

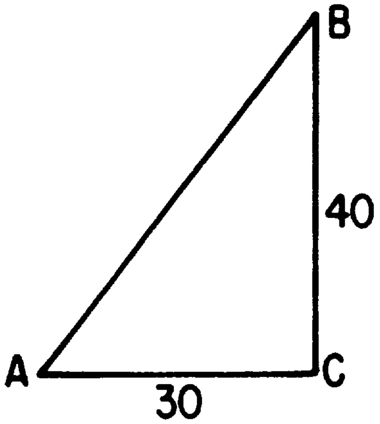

Figure 19-17.—Solving problems with a 3-4-5 triangle.

2. A guy wire 15 feet long is stretched from the top of a pole to a point on the ground 9 feet from the base of the pole. Calculate the height of the pole.

Answers:

1. 50 feet
2. 12 feet

OBLIQUE TRIANGLES

Oblique triangles were defined in chapter 17 of this training course as triangles which contain no right angles. A natural approach to the solution of problems involving oblique triangles is to construct perpendicular lines and form right triangles which subdivide the original triangle. Then the problem is solved by the usual methods for right triangles.

DIVISION INTO RIGHT TRIANGLES

The oblique triangle ABC in figure 19-18 has been divided into two right triangles by drawing line BD perpendicular to AC. The length of AC is found as follows:

1. Find the length of AD.

$$\frac{AD}{35} = \cos 40^\circ$$

$$AD = 35 \cos 40^\circ$$

$$= 35\ (0.7660)$$

$$= 26.8 \text{ (approximately)}$$

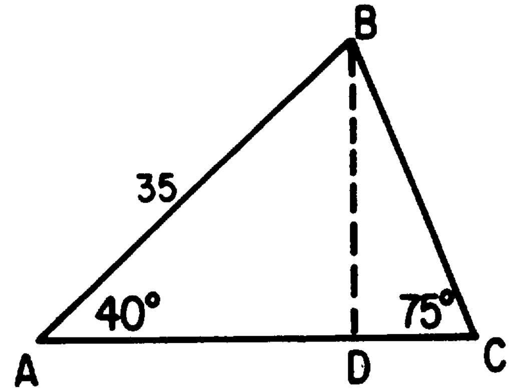

Figure 19-18.—Finding the unknown parts of an oblique triangle.

CAUTION: A careless appraisal of this problem may lead the unwary trainee to represent the ratio AC/AB as the cosine of 40°. This error is avoided only by the realization that the trigonometric ratios are based on RIGHT triangles.

2. In order to find the length of DC, first calculate BD.

$$\frac{BD}{35} = \sin 40^\circ$$

$$BD = 35 \sin 40^\circ$$

$$= 35\ (0.6428)$$

$$= 22.4 \text{ (approximately)}$$

3. Find the length of DC

$$\frac{22.4}{DC} = \tan 75^\circ$$

$$DC = \frac{22.4}{\tan 75^\circ} = \frac{22.4}{3.732}$$

$$DC = 6.01 \text{ (approximately)}$$

4. Add AD and DC to find AC.

$$26.8 + 6.01 = 32.81$$

$$AC = 32.8 \text{ (approximately)}$$

SOLUTION BY SIMULTANEOUS EQUATIONS

A typical problem in trigonometry is the determination of the height of a point such as B in figure 19-19.

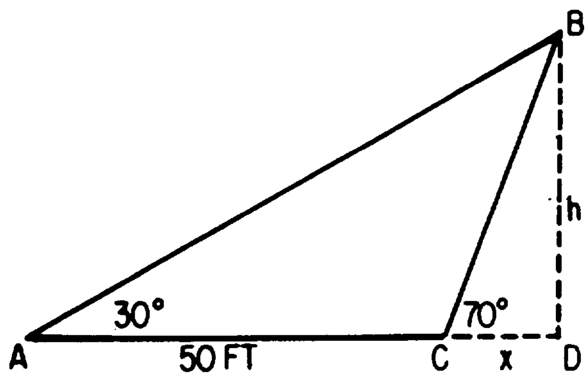

Figure 19-19.—Calculation of unknown quantities by means of oblique triangles.

Suppose that point B is the top of a hill, and point D is inaccessible. Then the only measurements possible on the ground are those shown in figure 19-19. If we let h represent BD and x represent CD, we can set up the following system of simultaneous equations:

$$\frac{h}{x} = \tan 70^\circ$$

$$\frac{h}{50 + x} = \tan 30^\circ$$

Solving these two equations for h in terms of x, we have

$$h = x \tan 70^\circ$$

and

$$h = (50 + x) \tan 30^\circ$$

Since the two quantities which are both equal to h must be equal to each other, we have

$$x \tan 70^\circ = (50 + x) \tan 30^\circ$$

$$x\ (2.748) = 50\ (0.5774) + x(0.5774)$$

$$x\ (2.748) - x\ (0.5774) = 28.8$$

$$x\ (2.171) = 28.8$$

$$x = \frac{28.8}{2.171} = 13.3 \text{ feet}$$

Knowing the value of x, it is now possible to compute h as follows:

$$\begin{aligned} h &= x \tan 70^\circ \\ &= 13.3\ (2.748) \\ &= 36.5 \text{ feet (approximately)} \end{aligned}$$

Practice problems:

1. Find the length of side BC in figure 19-20 (A).

2. Find the height of point B above line AD in figure 19-20 (B).

Answers:

1. 21.3 feet
2. 41.7 feet

LAW OF SINES

The law of sines provides a direct approach to the solution of oblique triangles, avoiding the necessity of subdividing into right triangles. Let the triangle in figure 19-21 (A) represent any oblique triangle with all of its angles acute.

The labels used in figure 19-21 are standardized. The small letter a is used for the side opposite angle A; small b is opposite angle B; small c is opposite angle C.

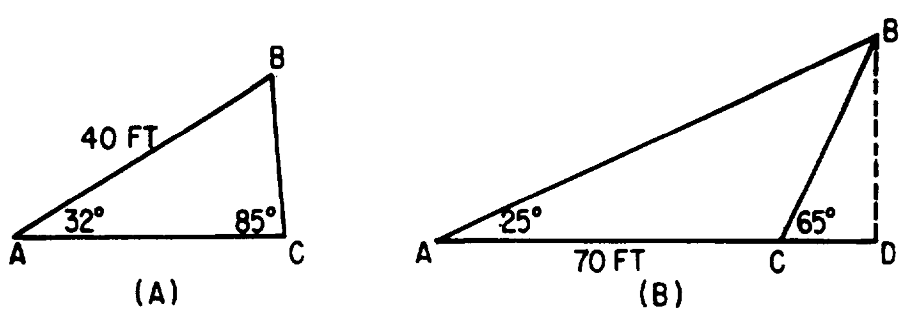

Figure 19-20.—(A) Oblique triangle with all angles acute; (B) obtuse triangle.

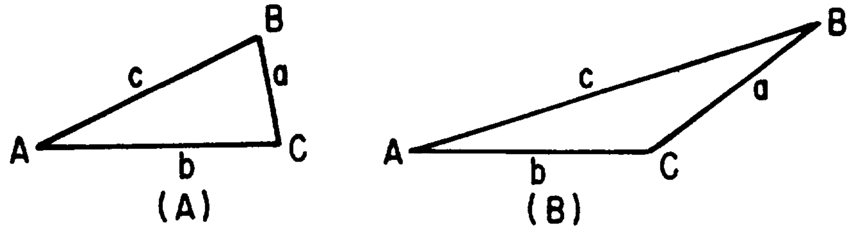

Figure 19-21.—(A) Acute oblique triangle with standard labels; (B) obtuse triangle with standard labels.

The law of sines states that in any triangle, whether it is acute as in figure 19-21 (A) or obtuse as in figure 19-21 (B), the following is true:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

EXAMPLE: In figure 19-21 (A), let angle A be 15° and let angle C be 85°. If BC is 20 units, find the length of AB.

SOLUTION: By the law of sines,

$$\frac{20}{\sin 15^\circ} = \frac{c}{\sin 85^\circ}$$

$$c = \frac{20 \sin 85^\circ}{\sin 15^\circ}$$

$$c = \frac{20\ (0.9962)}{0.2588} = 77.0$$